A Primer on the Cost of Capital for Banks

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ABSTRACT

The cost of capital is a key element in understanding the effect of capital requirements. We present a coherent cost of capital framework for banks based on the conservation of risk principle. As part of the framework, we consider a number of frictions commonly faced by banks, e.g., corporate taxes, government support, and a leverage ratchet effect. We illustrate the framework by providing a small collection of numerical examples. Finally, we discuss and illustrate a number of misconceptions when it comes to the effect of higher capital requirements.

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1. Introduction

This primer explains and derives the cost of capital for banks based on the conservation of risk principle. Conservation of risk is the principle underlying the analysis presented in Modigliani and Miller (1958). However, it is important to note that it does not imply that capital structure is irrelevant. The principle can best be thought of in terms of portfolios. The bank generates a cash flow, which can be distributed between equity and debt holders. The variability of the cash flow determines it's risk level and thereby the required rate of return. Once the cash flow is distributed between debt holders and equity holders, one can think of an investor holding a portfolio containing all debt and equity issued by the bank. This portfolio would then receive the total cash flow generated by the bank. The riskiness of the portfolio cash flow would therefore be the same as before it was distributed to the investor because the cash flows are identical when summed up. The riskiness of the total cash flow generated is thus conserved in the portfolio held by the investor. The riskiness of the cash flow does not change in the process of being distributed to the investor.

The conservation of risk principle is discussed in the Nobel Lecture of Miller (1990) and is also explicitly discussed for banks in Kashyap et al. (2010). The cost of capital calculations presented in this primer is based on the derivations in Dick-Nielsen et al. (2022), which also provides empirical support for the conservation of risk principle for banks. The cost of capital calculations is for most part similar to those for other types of firms and they are standard in the Corporate Finance literature. We first consider a bank without frictions and then introduce the most common frictions faced by banks. These are taxes, explicit government support, implicit government support, and a leverage ratchet effect.

For completeness, we illustrate what the cost of capital would look like in case the conservation of risk principle is violated. As part of the discussion, we explain why we consider this to be a very unlikely situation because it entails a disconnect between risks and returns. As part of the discussion, we also present a commonly used line of argumentation, which is supposed to illustrate that capital requirements are expensive for the customers of a bank. This calculation is, however, incoherent and we discuss the fallacies used in the argumentation. Capital requirements can indeed be costly for the equity holders of a bank but the fallacies lead to a substantial overestimation. Furthermore, the cost born by the equity holders is a private cost and not necessarily a social cost as we explain in the section on the effect of reducing leverage.

The primer ends with some numerical examples which illustrate the various cases and frictions with concrete numbers.

The primer proceeds as follows. Section 2 presents a coherent framework for the cost of capital for banks based on the conservation of risk principle. We look at an all-equity financed bank, a levered bank, and then introduce corporate taxes and government support. In Section 3 we show how the cost of capital changes when leverage is reduced and what implications this has. We furthermore introduce a leverage ratchet effect. In Section 4 we show for completeness what the cost of capital calculations could look like if conservation of risk is violated. In Section 5 we show a line of argumentation supposed to illustrate that capital requirements are costly. We then correct the incoherent calculations and discuss the fallacies impacting the argumentation. Finally, in the last section, we show a number of numerical examples.

2. Cost of Capital with Conservation of Risk

In this section we demonstrate how cost of capital for banks behave under the conservation of risk principle. We first consider debt and equity for a bank facing no market frictions. Afterwards, we show how frictions in the form of taxes and government subsidies affect the cost of capital.

We carefully spell out where the conservation of risk principle is used in the argumentation. Having this argumentation in place makes it easier to consider the effect of possible violations of the principle in later sections.

A. No Frictions

A.1. Unlevered Equity

Consider an all-equity financed bank with earnings (before interests and taxes) given as $EBIT$. We assume for simplicity that all cash flows are perpetual and given on a yearly basis with a constant expectation (we suppress a time t index on cash flows for convenience). The expected return r^A on the $EBIT$ cash flow is determined by the level of asset risk in the bank. Another way of saying this is that the $EBIT$ cash flow is derived from the assets of the bank and that this cash flow is uncertain. The possible future variation in realized cash flow posses a risk that investors would like to be compensated for carrying. This compensation can be mapped into the required rate of return that would satisfy the investors for carrying the given level of risk.

Let V^U be the expected discounted value of total cash flows available to investors (unlevered asset value) so that

$$V^U = \frac{\text{EBIT}}{r^A}. \tag{1}$$

We define the *total cost of capital* as the rate of return r^{total} required by an investor holding all the securities issued by the bank. In the all-equity financed bank, equity receives all available cash flows generated by the assets of the bank. The risk of the equity, i.e., all securities issued by the bank, is thus equal to the risk of the $EBIT$ cash flow given that conservation of risk holds true. Hence, $r^{total} = r^E = r^A$, where r^E is the cost of equity capital. By extension we also have

$$V^U = E, \tag{2}$$

meaning that the market value of equity is equal to the discounted available cash flows generated by the assets of the bank. While the latter seems trivial it rests on the assumption that conservation of risk holds true so that the risk of the generated cash flows is the same as (i.e. conserved in) the risk of the portfolio of securities issued by the bank. This means that the risk of the $EBIT$ cash flow does not change once it is distributed to equity holders.

A.2. Levered Equity

Next, assume that the bank is financed using both equity and debt. The market value of debt is D with a cost of debt equal to r^D. We take these debt characteristics as exogenously given. In the simplest case debt would be risk free and the cost of debt would then be equal to the risk free rate but this is not a necessary condition. The underlying assets are assumed to be the same as for the all-equity financed bank and $EBIT$ is therefore the same as before. The expected value of total discounted cash flows available to investors from the unlevered bank is called V^L.

The total cash flow available to investors can be calculated by first looking at the debt cash flow. The yearly expected interest payment is $r^D \times D$ and by construction the value of the debt satisfies that it can be calculated as expected discounted interest payments so that $D = (r^D \times D)/r^D$. The yearly residual cash flow available to equity holders is therefore $EBIT - r^D \times D$. The total cash flow to investors (both debt holders and equity holders) is therefore still equal to $EBIT$. Since the assets are the same as before, the asset risk is also the same as before. The discounted value of the total cash flow V^L for the levered bank is therefore equal to that of the unlevered bank, $V^L = V^U$. The total cash flow is unchanged and the cash flow risk is unchanged when the bank is financed by debt without any frictions.

As before we define the *total cost of capital* as the rate of return an investor requires for holding all the securities issued by the bank. Hence, we have that

$$r^{total-L} = \frac{E}{E+D} \times r^E + \frac{D}{E+D} \times r^D, \tag{3}$$

where $r^{total-L}$ is the total cost of capital for the levered bank. The total dollar return on the security portfolio is equal to the cash flows received by equity plus debt given as

$$r^E \times E + r^D \times D = \text{EBIT} - r^D \times D + r^D \times D = \text{EBIT}, \tag{4}$$

which is (trivially) equal to the total cash flow received by investors as already calculated. If we assume that conservation of risk holds, then as the total cash flow is identical to that of the unlevered bank and the security portfolio risk should not depend on how this total cash flow is distributed between debt and equity, i.e. how investors receive the total cash flow, we have that $V^L = E + D$ and

$$r^A = r^{total} = r^{total-L} = \frac{E}{E+D} \times r^E + \frac{D}{E+D} \times r^D. \tag{5}$$

The value of the levered bank without frictions is identical to the value of the unlevered bank. The total risk of the bank is also not dependent upon the capital structure choice.

We can isolate the cost of equity capital

$$r^E = r^A + (r^A - r^D)\frac{D}{E}. \tag{6}$$

As the debt to equity ratio is increased, the cost of equity will increase because equity as the residual claim becomes more risky. These results are standard for the Corporate Finance literature without frictions and can be seen as a variation of the Modigliani and Miller (1958) theorem on capital structure irrelevance.

When we later consider a debt ratchet effect as known from Admati et al. (2018) it will be convenient to also have book values in addition to market values. Let $\bar{D}$ be the book value of debt and $\bar{E}$ be the book value of equity. The book asset value $\bar{V}$ is then given as

$$\bar{V} = \bar{E} + \bar{D}. \tag{7}$$

The cost of capital is defined in terms of return on market values. There need not be any endogenous relationship between book values and market values. Book values are therefore not directly relevant for cost of capital considerations. However, we will use book values to later illustrate the potential market value loss for the original equity holders when retiring debt and financing this event by issuing new equity.

B. Corporate Taxes

B.1. Unlevered Equity with Corporate Taxes

Consider, again, an all-equity financed bank with earnings before interests and taxes given as $EBIT$ but now with a corporate tax rate of τ. Let V^{UwT} be the expected discounted value of total cash flows available to investors (unlevered asset value) so that

$$V^{UwT} = \frac{\text{EBIT} - \text{TAX}^U}{r^A} = \frac{\text{EBIT} \times (1-\tau)}{r^A} \tag{8}$$

where the yearly tax payment is given as $\text{TAX}^U = \tau \times \text{EBIT}$. Note that the tax payment will vary in the same way as $EBIT$ and therefore have the same level of risk and the same discount rate. This is assuming that $EBIT$ stays positive or that the tax code is somehow not dependent on the sign of $EBIT$.

The risk of the equity, i.e., all securities issued by the bank, is equal to the risk of the after-tax $EBIT$ cash flow given that conservation of risk holds true. Hence, $r^{total} = r^E = r^A$, where r^E is the cost of equity capital. By extension we also have

$$V^{UwT} = E, \tag{9}$$

meaning that the market value of equity is equal to the discounted after-tax available cash flows generated by the assets of the bank.

The introduction of corporate taxes for the unlevered bank essentially just scales the value of the bank downwards, but does not change the riskiness of the bank and therefore does not change the total cost of capital.

To provide a numerical example we make the following assumptions:

- The bank is funded using 1,000,000 in equity (Book value)
- Total assets are therefore 1,000,000 (Book value)
- Earnings before interest payments and taxes (EBIT) are 70,000 per year.
- Required return on assets r^{total} is 5%
- The tax rate is 26%

Given these assumptions the bank balance sheet in book values becomes:

Bank Balance Sheet (Book values)

Assets = 1,000,000	Equity = 1,000,000

We can then find the market value of the bank by first calculating earnings after taxes and then discount the cash flows.

- Earnings after taxes are $(1-0.26) \times 70,000 = 51,800$.

- Market value of assets and of equity then becomes $51,800/0.05 = 1,036,000$.

Note that the balance sheet in book values did not impact the market value calculation in this example. Book values will not really matter until we consider the leverage ratchet effect in a later section.

B.2. Levered Equity with Corporate Taxes

The bank is now financed using both equity and debt and is subject to corporate taxes. The market value of debt is D with a cost of debt equal to r^D. The underlying assets are still assumed to be the same as in the previous examples and $EBIT$ is therefore the same as before. The value of total discounted cash flows available to investors from the unlevered bank is called V^{LwT}.

The total cash flow available to investors can be calculated by first looking at the debt cash flow. The yearly expected interest payment is $r^D \times D$ and by construction the value of the debt again satisfies that $D = (r^D \times D)/r^D$. Interest payments reduce the total tax payment (assuming earnings after interest but before taxes (EBT) are positive). The yearly expected tax payment is therefore reduced by the interest payment so that $\text{TAX}^L = \tau \times (\text{EBIT} - r^D \times D) = \tau \times \text{EBT}$. The yearly after-tax cash flow available to investors is therefore

$$EBIT - \text{TAX}^L = EBIT - \tau \times (\text{EBIT} - r^D \times D) = EBIT \times (1-\tau) + r^D \times D \times \tau. \tag{10}$$

The risk of the first part of the cash flow is the asset risk as for the unlevered bank since it is the exact same cash flow. The second term is the tax shield. The risk of the tax shield is also equal to the unlevered asset risk when the bank follows a constant leverage ratio strategy (DeMarzo, 2005). This assumption is empirically supported for banks by the findings presented in Dick-Nielsen et al. (2022). The discount rate for both cash flow parts should thus be equal to the unlevered asset risk r^A and V^{LwT} can then be found as

$$V^{LwT} = PV(EBIT - \text{TAX}^L) \tag{11}$$

$$= \frac{\text{EBIT} \times (1-\tau)}{r^A} + \frac{r^D \times D \times \tau}{r^A} = V^{UwT} + PV(\text{Tax Shield}). \tag{12}$$

Hence, we have that due to the tax shield, the value of the levered bank subject to corporate taxes measured in discounted cash-flow terms is higher than for the unlevered bank subject to corporate taxes.

Unsurprisingly, the value of the levered bank subject to corporate taxes is lower than for the levered bank not subject to corporate taxes. Tax payments naturally reduce the value of the bank. However, as shown above, the market value of the unlevered bank subject to taxes is also lower than the levered bank subject to taxes due to the tax shield from interest payments. This is the classical result from Miller (1977) that when taxes are the only friction and firms receive a tax deduction from interest payments there is an incentive to take as much debt financing as possible.

As before the *total cost of capital* is the rate of return an investor requires for holding all the securities issued by the bank. Hence, we have that

$$r^{total-LwT} = \frac{E}{E+D} \times r^E + \frac{D}{E+D} \times r^D, \tag{13}$$

where $r^{total-LwT}$ is then the total cost of capital for the levered bank subject to corporate taxes. Equity will get the residual cash flow after taxes and after interest payments and will receive

$$\text{EBIT} - r^D \times D - \tau \times (\text{EBIT} - r^D \times D) \tag{14}$$

which is EBIT minus interest payments and minus the tax payment. The total dollar return on the security portfolio is equal to the cash flows received by equity plus debt given as

$$r^E \times E + r^D \times D = \text{EBIT} - r^D \times D - \tau \times (\text{EBIT} - r^D \times D) + r^D \times D \tag{15}$$

$$= \text{EBIT} \times (1 - \tau) + r^D \times D \times \tau, \tag{16}$$

which is (trivially) equal to the total cash flow collectively received by investors as already calculated in Equation (10). If we assume conservation of risk holds true, then as the cash flows are identical, the security portfolio risk should not depend on how the cash flow is distributed between debt and equity. Hence, we have that $V^{LwT} = E + D$ and that

$$r^A = r^{total} = r^{total-LwT} = \frac{E}{E+D} \times r^E + \frac{D}{E+D} \times r^D. \tag{17}$$

Note that the risk of the levered and the unlevered bank does not depend on the corporate taxation because we assume that the bank follows a constant leverage ratio strategy. The same is true for the cost of equity. If we isolate the cost of equity, we get

$$r^E = r^A + (r^A - r^D)\frac{D}{E}, \tag{18}$$

which is identical to the cost of equity for the levered bank without taxes. Taxation thus affects the value of the bank but not the riskiness and therefore not the cost of capital.

To provide a numerical example we make the following assumptions:

- The bank is funded using 150,000 in equity and 850,000 in Debt (Book value). The debt is issued at par value, so that market value and book value are identical for debt.
- Total asset value is 1,000,000 (Book value).
- Earnings before interest payments and taxes (EBIT) are 70,000 per year.
- Required return on assets r^{total} is 5%.
- Required return on debt is 3.05%.
- The tax rate is 26%.

Given these assumptions the bank balance sheet is:

Bank balance sheet (Book values)

Assets = 1,000,000	Debt = 850,000
	Equity = 150,000

With these assumptions, we can calculate the market value of equity and the total value of the bank.

- Interest payments are $0.0305 \times 850,000 = 25,925$.
- Earnings after interest payments (EBT) but before taxes are $70,000 - 25,925 = 44,075$.
- Earnings after taxes and interest payments are $(1 - 0.26) \times 44,075 = 32,515.5$.

- The expected present value of the tax shield is $(0.0305 \times 850,000 \times 0.26)/0.05 = 134,810$.

- The market value of the bank can be found as the market value of the bank where we ignore the tax deduction from interest rates plus the value of the tax shield $1,036,000 + 134,810 = 1,170,810$. An alternative way to calculate this is discount the total cash flow returned to investors. We sum interest payments and earnings paid to equity holders and discount by the total required rate of return $(25,925 + 32,615.5)/0.05 = 1,170,810$.

- Using Equation (18) the return on equity becomes
  $0.05 + (0.05 - 0.0305) \times (850.000/(1,117,810 - 850,000)) = 0.1017$.
  An alternative way to calculate this is to find the ratio of earnings paid to equity holders over total equity value $32,615.5/(1,170,810 - 850,000) = 0.1017$.

Note that as shown above, the market value of the levered bank is the same as the market value of the unlevered bank plus the value of the tax shield.

C. Explicit Government Support

Government support can come in many forms, e.g., the tax deduction from interest payments resulting in a tax shield. Here we consider a government guarantee on bank debt in return for a fee. In case the bank defaults on it's debt obligations then the government explicitly promises to step in and paid debtors back in full. The situation thus resembles a depositor insurance scheme.

C.1. Fair Fee

We first consider a situation where the government explicitly guarantees the debt and the bank has to paid a fair fee. The assets are the same as in earlier examples and the bank is funded by equity and debt. The expected cash flow available to investors now has three components. First, the $EBIT$ cash flow from operations. Second, the expected government payment in default, and, third, the fee payment. In prior examples, we have assumed that cash flows could be represented as constant expected perpetual payments. However, this is not convenient for a payment in default and it is also not a necessary condition. So far we have only used this assumption for illustration. Here we will instead directly look at present values of the cash flows without specifying a certain structure for the individual expected cash flows and we only specify cash flow structures where it becomes convenient for illustration.

The expected present value of the cash flow available to investors is given as

$$V^{GS} = PV(\text{EBIT}) + PV(GS) - PV(\text{Fee}) \tag{19}$$

$$= V^U + GS - FF, \tag{20}$$

where $PV(GS)$ is the present value of the payment from the government in default, which we call GS. The present value of the fee payment is PV(Fee), which we call FF. Finally, the $EBIT$ cash flow has the same riskiness as the unlevered assets and so the expected present value of the available cash flows is the unlevered asset value plus the present value of the government support, and minus the present value of the fee payment.

In an actuarially fair insurance contract the expected fee payment is equal to the expected payment in default, so that the present value of the contract is zero. For our example, this would mean that $GS = FF$. The expected cash flow value then becomes $V^{GS} = V^{U}$. The total cost of capital can then be written as

$$r^{\text{total-GS}} = r^{A} \times \frac{V^{U}}{V^{GS}} + r^{GS} \times \frac{GS}{V^{GS}} - r^{FF} \times \frac{FF}{V^{GS}} \tag{21}$$

$$= r^{A} + (r^{GS} - r^{FF}) \times \frac{GS}{V^{GS}}. \tag{22}$$

The expected return of the government support payment r^{GS} is likely very low. Banks most often default when the market as a whole is during poorly. This means that the payment will come in bad times and so the correlation with the market return is likely negative. Investors are therefore willing to accept an expected return which might be lower than the risk free rate, $r^{GS} < r^{rf}$. This is also known from insurance contracts in general and can also be compared to deep-out-of-the-money put options. Next, consider the expected return on the fee payment. We can assume that the fee payment is a fixed yearly payment made from the bank to the government in all states-of-the-world where the bank has not yet defaulted. Once the bank defaults, the fee payment stops and the bank ceases to exist. This means that the fee payment resembles the payment from a perpetual debt contract without recovery. For the recipient, the expected return of the fee payment is thus higher than for debt issued by the same bank but without government support. This is because regular debt would have a recovery payment to debtors in default. We benchmark the bank to a similar bank without government support. The banks have the same underlying assets and are financed by the same market value of debt. The benchmark bank is then a classical

Modigliani-Miller bank as in Section 2.A.2. For the benchmark bank, we write

$$r^A = r^{E-MM} \times \frac{E^{MM}}{V^U} + r^D \times \frac{D}{V^U}. \tag{23}$$

With the benchmark bank in place, we can write that $r^{FF} > r^D$, because the fee payment cash flow is riskier than a regular bond issued by the benchmark firm. Going bank to the total cost of capital for the bank with government support as stated in Equation (22), we note that $r^{GS} - r^{FF} < 0$. A bank with higher level of debt will most likely have a higher value of GS because it becomes more likely that the bank will default and that the government will have to step in. Higher debt therefore puts more weight onto the second term of the equation. As the second term has a negative coefficient, the effect on the total cost of capital is that it decreases as debt increases. The total value of the bank does not change, but the total riskiness of the bank decreases with more debt due to the presence of a fairly priced government guarantee. This is as exactly as one would expect from a fair insurance contract; it does not add value but reduces overall risk.

Next, consider what happens to the cost of equity capital (still assuming conservation of risk). We first note that debt now becomes risk free from the perspective of the investor. As the government steps in and repays debt in default, debt investors will never experience a loss on their investment. The cost of debt therefore becomes the risk free rate r^{rf}. Comparing to the benchmark bank with the same market value of debt there will be a difference in interest payments. The interest payments for the benchmark bank is $r^D \times D$ and for the bank with government support it becomes $r^{rf} \times D$. The difference in interest payments constitutes an additional cash flow going to equity holders (until default) for the bank with government support. This additional yearly cash flow to equity holders is equal to $(r^D - r^{rf}) \times D > 0$. Note that this is a constant fixed additional payment until default and it becomes zero in default. This additional cash flow therefore has the same level of risk as the fee payment and the same expected return of r^{FF}. Note also that because the fee is assumed to be fair the size of the fee payment is exactly equal to this additional payment to equity holders so that the yearly fee payment is $\text{Fee} = (r^D - r^{rf}) \times D$. The residual payment to equity holders therefore becomes

$$EBIT - r^D \times D + (r^D - r^{rf}) \times D - \text{Fee} \tag{24}$$

$$= EBIT - r^D \times D, \tag{25}$$

where the first two terms is the same cash flow as that going to equity holders in the benchmark bank. The third term is the additional cash flow available to equity holders in the bank with government support coming from the lower interest payments and the fourth term is the fair fee payment. Inserting the fair fee payment then reduces the cash flow to match that of the benchmark bank. Since the underlying assets are the same for the two banks, then the value of the cash flow is also the same. We therefore have that $E^{MM} = E^{GS}$ and that $r^{E-MM} = r^{E-GS}$, where E^{GS} is the equity value in the bank receiving government support in return for a fair fee and r^{E-GS} is the cost of equity capital for the same bank.

Summarizing the findings, we find that explicit government support in return for a fair fee paid by the bank does not affect the total value of the bank. It does, however, reduce the total risk of the bank similar to other insurance contracts. The cost of debt decreases to the risk free rate but the cost of equity is the same as for the benchmark bank without government support and the value of the equity is also not affected by the explicit fairly priced government support.

C.2. Partial Fee

The fee payment for participating in depositor insurance schemes are sometimes said to be lower than the fair fee. We therefore consider a situation where the fee payment is only a fraction of the government support value. Let now $FF_\gamma = \gamma \times GS$, where $0 \leq \gamma \leq 1$ and FF_γ is the present value of the partial fee payment. The expected value of the available cash flows to all investors then becomes

$$V^{GS\gamma} = PV(\text{EBIT}) + PV(GS) - PV(\text{Partial Fee}) \tag{26}$$

$$= V^U + GS - FF_\gamma \tag{27}$$

$$= V^U + (1-\gamma)GS. \tag{28}$$

When the fee payment is only partial and thus smaller than the expected value of the government support, we can see that the expected value of the available cash flow (the value of the bank with conservation of risk) becomes larger than for the benchmark bank. As the level of debt increases, the value of the government support becomes larger, and the value of the bank therefore also becomes larger. The government is essentially bestowing the bank with an implicit subsidy for free. The total cost of capital now becomes

$$r^{\text{total-GS}\gamma} = r^A \times \frac{V^U}{V^{GS\gamma}} + r^{GS} \times \frac{GS}{V^{GS\gamma}} - r^{FF} \times \frac{FF_\gamma}{V^{GS\gamma}} \tag{29}$$

$$= r^A \times \frac{V^U}{V^{GS\gamma}} + (r^{GS} - \gamma \times r^{FF}) \times \frac{GS}{V^{GS\gamma}} \tag{30}$$

$$= r^A \times \frac{V^U}{V^{GS\gamma}} + (r^{GS} - r^{FF}) \times \frac{GS}{V^{GS\gamma}} + (1-\gamma) \times r^{FF} \times \frac{GS}{V^{GS\gamma}}. \tag{31}$$

Note first that while the fee payment is now only a fraction of the fair payment it still has the same construction and the same level of risk as the fair payment. It is a fixed payment until default and zero thereafter. The bank also defaults at the same point in time as the bank with the fair fee because the interest payments are the same. The expected return on the partial fee payment is therefore the same as for the fair fee payment r^{FF}. In the final line of the equation, we have set up the formula so that the first two terms resemble the case for the bank with the fair fee and the partial fee then gives rise to the last term in the equation. It is reasonable to assume that the asset risk is higher than the risk of the fee payment so that $r^A > r^{FF}$. This means that as the debt increases the expected value of the government support will increase, more weight will then be put on the last term of the equation, and the total cost of capital will therefore be higher than for the bank with a fair fee payment. This is because more weight will be put on the second term which decreases the total cost of capital but this effect will then be dampened by the last term because $r^{GS} - r^{FF} < 0 < (1-\gamma) \times r^{FF}$. The intuition is that for the bank with the fair fee, higher debt will make the total cost of capital tend to the risk free rate when the bank is all debt financed. For the bank with a partial fee, the bank will still get an additional risky cash flow from the reduced fee until default. However, since $r^A > (1-\gamma) \times r^{FF}$ the total cost of capital for the bank with partial fee payment will still be smaller than for the Modigliani-Miller benchmark bank.

Next, we again consider what happens to the cost of equity capital and the value of equity capital. We use the Modigliani-Miller bank as benchmark again and write up the total value of the bank with partial fee payment where we substitute in the value of the benchmark bank

$$V^{GS\gamma} = E^{GS\gamma} + D \tag{32}$$

$$= V^U + (1-\gamma)GS \tag{33}$$

$$= E^{MM} + D + (1-\gamma)GS. \tag{34}$$

Reducing the equation gives that $E^{GS\gamma} = E^{MM} + (1-\gamma)GS$, meaning that the added value from the partial fee payment goes directly to equity holders. This value transfer from the government to equity holders is increasing in the level of debt as higher debt makes the government support GS more valuable.

The cost of debt is still the risk free rate r^{rf}, but the cost of equity has now changed. For equity we can write the dollar return as

$$r^{E-GS\gamma} \times E^{GS\gamma} = r^{E-MM} \times E^{MM} + r^{FF} \times (1-\gamma)GS. \tag{35}$$

Here we have used the value equation for the equity in the firm with partial fee payment. It may not be obvious that the expected return of the second term in the equation is r^{FF}, however, remember that the additional equity value compared to the benchmark bank comes from the reduced interest payment, which is now only partially consumed by the fee payment. The extra value thus derives from a fixed cash flow until default and the riskiness of this cash flow is therefore equal to that of the fee payment as argued earlier. Isolating the cost of equity gives

$$r^{E-GS\gamma} = r^{E-MM} \times \frac{E^{MM}}{E^{GS\gamma}} + r^{FF} \times \frac{(1-\gamma)GS}{E^{GS\gamma}} \tag{36}$$

$$= r^{E-MM} \times \omega + r^{FF} \times (1-\omega), \tag{37}$$

where we have inserted ω to illustrate that the fractions become portfolio weights summing to 1. As debt increases more weight is put onto the second term and since $r^{E-MM} > r^{FF}$ the cost of equity capital will decrease compared to the benchmark bank's cost of equity.

Summarizing for the bank with government support and a partial fee payment; the value of the bank will increase compared to the benchmark bank and to the bank paying a fair fee. The riskiness of the bank will be in between the riskiness of these two banks with the fair fee bank having the lowest risk. The partial fee payment will constitute a value transfer from the government to the bank. This value transfer will go to the equity holders of the bank and both increase equity value and reduce equity risk.

D. Implicit Government Support

After having investigated explicit government support as is known from, e.g., depositor insurance programs, it is also relevant to consider implicit government support. This could be expected support from a too-big-to-fail guarantee. A bank, which investors expect to be too-big-to-fail, will be able to fund themselves with low cost debt. Hence, we consider a bank for which investors expect that in the case of default the government will step in and repay debt investors in full. Assuming that this expectation is very strong, investors are willing to accept an interest rate equal to the risk free rate on the debt issued by the bank. The situation then becomes identical to the situation analyzed for the bank paying only a partial fee for government support as in Section 2.C.2, except that $\gamma = 0$.

The result of a too-big-to-fail guarantee in this form will then be that the value of the bank will be higher than for the benchmark bank and the riskiness will be in between that of the benchmark bank and the bank paying a fair fee. The value of equity will be higher than for the benchmark bank and the bank paying any kind of non-zero fee. The riskiness of equity will also become lower than for these two cases.

3. Reducing Market Leverage

When discussing the cost of higher capital requirements it is helpful to consider what happens when market leverage is reducing in a setting with conservation of risk. For the bank with no frictions and conservation of risk holding true nothing will happen to the total cost of capital or total market value. Here, we instead analyze the case with two frictions, namely leverage reduction in the presence of corporate taxes and with a leverage ratchet effect.

A. Reduction with Corporate Taxes

Assume that a bank financed with debt and equity and subject to corporate taxes (same setup as in Section 2.B.2) reduces market leverage from $\frac{D_1}{V_1}$ to $\frac{D_2}{V_2}$. This reduction is obtained by reducing the market value of debt D so that $D_1 > D_2$. The underlying assets are still the same as for the unlevered bank and the risk of the $EBIT$ cash flow is therefore unchanged from prior examples. After the debt reduction, the bank continues to follow a constant leverage ratio policy but now at a new target ratio and the risk of the tax shield is therefore still equal to the risk of the unlevered assets. The risk of the cash flows generated by the levered bank is therefore the same both before and after the reduction in leverage and equal to the risk of the unlevered cash flow r^A. This means that the discounted expected values of the available cash flows before and after the debt reduction can be written as

$$V_1^L = \frac{\text{EBIT} \times (1-\tau)}{r^A} + \frac{r_1^D \times D_1 \times \tau}{r^A} = V^U + PV(\text{tax shield}_1), \tag{38}$$

and

$$V_2^L = \frac{\text{EBIT} \times (1-\tau)}{r^A} + \frac{r_2^D \times D_2 \times \tau}{r^A} = V^U + PV(\text{tax shield}_2), \tag{39}$$

where

$$PV(\text{tax shield}_2) < PV(\text{tax shield}_1) \tag{40}$$

so that $V_2^L < V_1^L$. Hence, reducing leverage does not change the total cash flow risk and therefore the total cost of capital is unchanged but it reduces bank value due to a loss of tax shield.

Assuming conservation of risk and using the same argumentation as earlier we have that

$$V_2^L = E_2 + D_2 < E_1 + D_1 = V_1^L \tag{41}$$

The reduction in leverage may reduce credit risk resulting in a lower cost of debt. However, independent of the possible reduction in credit risk, lower leverage reduces equity risk despite the unchanged total cost of capital. This can be seen in the following way: Assuming conservation of risk, the following relationship holds true for a levered bank for all capital structures

$$r^A = \frac{E}{E+D} \times r^E + \frac{D}{E+D} \times r^D, \tag{42}$$

rearranging the terms

$$r^E = r^A + (r^A - r^D)\frac{D}{E}. \tag{43}$$

Since r^A does not change then lowering the leverage ratio reduces the second term and the cost of equity decreases.

Summarizing the effect of a reduction in leverage under conservation of risk and subject to corporate taxes, the value of the bank will decrease due to a loss of tax shield. Here it becomes relevant to note that this value loss is a private cost for the equity holders of the bank. It is not necessarily a social cost because the government will now receive a higher tax payment equal to the reduction in bank value. If the government can spend the additional tax revenue elsewhere as efficiently or more efficiently then there is no social cost despite the private cost for equity holders. The reduction in leverage likely cause the cost of debt to decrease and will in all cases reduce the cost of equity without affecting the overall cost of capital for the bank.

B. Reduction with a Leverage Ratchet Effect

The prior example did not make any distinction between new and old equity holders when issuing new equity. However, reducing debt by issuing new equity can be costly for the old equity owners if we assume a leverage ratchet effect (Admati et al., 2018). A leverage ratchet effect arises if current debt holders anticipate lower credit risk after a debt reduction and, hence, are unwilling to sell their debt back at the pre-credit risk reduction value. The selling debt holders need to be compensated for the foregone credit risk improvement and this compensation constitutes a cost for the current equity holders. It is important to note that this ratchet effect does not affect any calculation in prior examples and the ratchet effect can be there even in a pure Modigliani-Miller setup. The ratchet effect agency cost to current shareholders may help explain why existing equity holders are often unwilling to voluntarily retire debt, even though it does not affect the total market value of the bank.

In order to incorporate the leverage ratchet effect, we need to consider the book value of debt and the book value of equity. Assume that debt is issued at par value so that market value D_1 and book value $\bar{D}_1$ initially coincides, i.e. $D_1 = \bar{D}_1$. Assume for simplicity that the book value of equity $\bar{E}_1$ also initially equals the market value of equity E_1. We therefore have that

$$E_1 + D_1 = \bar{E}_1 + \bar{D}_1. \tag{44}$$

Assume again for simplicity that the reduction in debt is done by retiring all the old debt and issuing new debt at par value so that $D_2 = \bar{D}_2$. The book value of debt plus equity after this change in capital structure can be written as

$$\bar{V}_2^L = \bar{E}_{NEW} + \bar{E}_1 + \bar{D}_2, \tag{45}$$

so that the book value of the equity already in place $\bar{E}_1$ is unchanged and the new equity $\bar{E}_{NEW}$ is added to the balance sheet. The book value of debt has alsp changed from $\bar{D}_1$ to $\bar{D}_2$. The total book value of equity is thus now $\bar{E}_2 = \bar{E}_{NEW} + \bar{E}_1$ but because we need to be able to distinguish between new and old equity, we keep these separate.

The book value of the new equity enters the balance sheet at the market value and hence $\bar{E}_{NEW} = E_{NEW}$. In order to retire the old debt the bank needs raise equity in the amount of

$$E_{NEW} = \frac{r_1^D \times D_1}{r_2^D} - D_2 = \frac{r_1^D}{r_2^D} D_1 - D_2, \tag{46}$$

where the first term is the old interest payments discounted at the new cost of debt and the second term is the amount of funds raised by issuing new debt. The new total bank market value can now be written as the sum of the market value of the new equity, the market value of the old equity, and the market value of the new debt so that $V_2^L = E_{NEW} + E_{OLD} + D_2$. Substituting in the market value of the new equity gives us that

$$V_2^L = E_{NEW} + E_{OLD} + D_2 \tag{47}$$

$$= \frac{r_1^D}{r_2^D} D_1 - D_2 + E_{OLD} + D_2 \tag{48}$$

$$= \frac{r_1^D}{r_2^D} D_1 + E_{OLD}, \tag{49}$$

where we can assume that $r_1^D > r_2^D$, which corresponds to a situation where credit risk declines as leverage goes down, so that $\frac{r_1^D}{r_2^D} > 1$.

Let us first disregard the tax shield so that the market value of the bank is not affected by the change in capital structure. We then have that $V_1^L = V_L^2$ and since $V_1^L = E_1 + D_1$ we can write

$$E_1 + D_1 = \frac{r_1^D}{r_2^D} D_1 + E_{OLD}. \tag{50}$$

Isolating the new market value of the old equity gives that

$$E_{OLD} = E_1 - \left(\frac{r_1^D}{r_2^D} - 1 \right) D_1 < E_1. \tag{51}$$

Hence, the market value of equity held by the old share holders is now lower than before the change in capital structure. This reduction is due to the leverage ratchet effect.

We can easily incorporate the loss of tax shield along side the leverage ratchet effect. We then extend the bank market value formulas with the tax shield so that

$$V_1^L = V_2^L + PV(\text{Tax Shield Loss}), \tag{52}$$

where the present value of the tax shield loss is just the difference in tax shields before and after the reduction in debt as seen in the prior example. We then repeat the equation from before but now with the tax shield loss so that

$$V_1^L = E_1 + D_1 \tag{53}$$

$$= V_2^L + PV(\text{Tax Shield Loss}) \tag{54}$$

$$= E_{NEW} + E_{OLD} + D_2 + PV(\text{Tax Shield Loss}) \tag{55}$$

$$= \frac{r_1^D}{r_2^D} D_1 + E_{OLD} + PV(\text{Tax Shield Loss}) \tag{56}$$

Rearranging terms gives us that

$$E_{OLD} = E_1 - \left(\frac{r_1^D}{r_2^D} - 1 \right) D_1 - PV(\text{Tax Shield Loss}), \tag{57}$$

which shows that the old shareholders experience a value loss coming from the leverage ratchet effect and from the decrease in the tax shield value. Note, however, that the total bank value is only decreased by the tax shield loss and that the total cost of capital is as explained before the introduction of a ratchet effect. The leverage ratchet only constitutes a value transfer between new and old equity holders (and selling debt holders).

To provide a numerical example we now assume that the bank decides to buy back debt and finance it by issuing additional equity. Specifically we assume that the bank is asked to reduce the debt by $50,000$ and fund it via issuance of new equity.

Before the bank reduces debt the balance sheet is as in the previous numerical example:

Bank balance sheet (Book values)

Assets	Liabilities
Assets = 1,000,000	Debt = 850,000
	Equity = 150,000

The assumptions are as before:

- Earnings before interest payments and taxes (EBIT) are 70,000 per year.
- Required return on assets r^{total} is 5%
- Required return on debt before the reduction is 3.05%
- The tax rate is 26%

After the reduction in debt, we assume that:

- The required return on debt declines to 3.00%.
- To simplify calculations, we assume as described in the theory above, that the bank buys back all outstanding debt and issue new debt at par value. The par value of debt therefore becomes $800,000$ with an expected return of 3.00%. Note again, that this is not a necessary assumption but it simplifies the number of moving parts in the calculation.
- Earnings before interest payments and taxes (EBIT) are not affected by the change in capital structure because the asset side is left unchanged. Hence, EBIT is still $70,000$.

With these assumptions in place, we can find the market value of equity and the market value of the bank after the change in capital structure.

- Interest payments are $(0.03 \times 800,000) = 24,000$.
- Earnings after interest payments but before taxes (EBT) are $70,000 - 24,000 = 46,000$.
- Earnings after taxes and interest payments are $(1 - 0.26) \times 46,000 = 34,040$.
- The market value of the bank becomes $(24,000 + 34,040)/0.05 = 1,160,800$.
- The drop in market value compared to before the reduction in debt is $1,170,810 - 1,160,800 = 10,010$. This corresponds to the loss of tax shield value, which we could also have found as $(850,000 \times 0.0305 - 800,000 \times 0.03) \times 0.26/0.05 = 10,010$.
- The market value of equity before was $1,170,810 - 850,000 = 320,810$. The market value of equity after the change is $1,160,800 - 800,000 = 360,800$.

We can see that the market value of equity increases. This is because we keep EBIT unchanged but lower the interest payment so that the residual cash flow available to equity holders becomes larger. The return on equity can be found as:

- Before the change the return on equity was 0.1017.
- After the change the return on equity becomes $34,040/360,800 = 0.0943$.

Equity has become less risky and so the required return has become lower than before. So far we have not considered the effect of the leverage ratchet. But we can calculate how much new equity (in market values) that the bank needs to use in other to buy back the debt. We use Equation (46):

- Market value of the new equity $0.0305/0.030 \times 850,000 - 800,000 = 64,166.67$. We note that this is higher than the reduction in market value for the debt.
- The market value of equity held by the old equity holders become $360,800 - 64,166.67 = 296,633.3$.
- The drop in market value for the old equity holders then become $320,810 - 296,633.3 = 24,176.7$. This loss is a combination of the tax shield loss and the leverage ratchet effect.

The bank needs to issue new equity with a market value of $64,166.67$ in order to retire $50,000$ of debt. The new equity enters the balance sheet so that book value and market value of the new equity matches. The balance sheet after the change in capital structure:

Bank balance sheet (Book values)

Assets = 1,014,167	Debt = 800,000
	Equity = 214,166.7

We see that the bank ends up holding more capital than if there is no ratchet effect. The effect from the ratchet effect is the additional $14,166.67$ held in equity capital.

4. Violating Conservation of Risk

The calculations so far have been done assuming that conservation of risk holds true. This is consistent with the empirical findings in Dick-Nielsen et al. (2022). The vast majority of theoretical models also assume that conservation of risk holds true. However, one can also do the above calculations assuming that conservation of risk is violated. We do not consider this to be a realistic scenario but we here illustrate the calculations for reference.

When risk is not conserved from the available cash flows to the portfolio of securities issued by the bank then one can essentially assuming anything and get any result. We will illustrate the calculations for a scenario when leverage is reduced and where it is assumed that the cost of equity decreases by less than if risk conversation holds. Best we can tell, this is the most common example of a violation of the conservation of risk assumption.

Consider a bank financed by equity and debt using the same setup as in Section 3.A but without taxes for simplicity. We will assume that conservation of risk held true before the change in capital structure so that

$$V_1^L = E_1 + D_1 = V^U \tag{58}$$

and with the cost of debt given as r_1^D, the cost of equity satisfy

$$r_1^E = r^A + (r^A - r_1^D) \times \frac{D_1}{E_1}. \tag{59}$$

After a reduction in debt, the new cost of debt equals r_2^D and $r_1^D > r_2^D$. With conservation of risk the new cost of equity should have been

$$r_2^{E-conv} = r^A + (r^A - r_2^D) \times \frac{D_2}{E_2^{conv}} = r^A + (r^A - r_2^D) \times \frac{D_2}{V^U - D_2}, \tag{60}$$

but because conservation of risk is violated this is no longer true. In order to have some structure on the violation, we will assume that the new cost of equity lies between the old cost of equity r_1^E and the post-cost of equity without violations r_2^{E-conv}. We therefore model the new cost of equity as

$$r_2^E = r_1^E - \alpha \times (r_1^E - r_2^{E-conv}), \tag{61}$$

where α is then the fraction of the conservation of risk discount that the new equity receives. The value of debt after the reduction in debt is given as

$$D_2 = \frac{r_2^D \times D_2}{r_2^D}. \tag{62}$$

Hence, we value debt as the expected discounted interest payment and the value is therefore equal to D_2. The value of equity is now found as the cash flow to equity discounted with the cost of equity

$$E_2 = \frac{EBIT - r_2^D \times D_2}{r_2^E} \tag{63}$$

Because the discount rate is now higher than when conservation of risk holds true, the equity value will decrease compared to the benchmark case with no violation of conservation of risk. We can illustrate this further by looking at the dollar return on the combined portfolio of equity and debt

$$r_2^E \times E_2 + r_2^D \times D_2 = EBIT - r_2^D \times D_2 + r_2^D \times D_2 \tag{64}$$

$$= EBIT. \tag{65}$$

The total portfolio cash flow is still $EBIT$ because the assets have not changed. But for some reason r_2^E is higher than according to conservation of risk. But this means that E_2 has to decrease in order for the dollar return to still be equal to $EBIT$. This also implies that total bank risk as measured by the security portfolio risk has increased. To see this first note that $V_2^L = E_2 + D_2 < V_1^L$, so that the market value of the bank has now decreased because of the loss in equity value. But we also have that

$$r_2^{total} \times V_2^L = r_2^E \times E_2 + r_2^D \times D_2 = EBIT. \tag{66}$$

As V_2^L is lower than V_1^L, this shows , given the total dollar return is still equal to $EBIT$, that r_2^{total} has to increase.

This is inconsistent with conservation of risk in the following way. The operating cash flow available to distribution between equity and debt holders is equal to $EBIT$. This is the total cash flow distributed both before and after the capital structure change. This cash flow has a certain level of uncertainty derived from the assets that generate the cash flow. These assets have not changed. Hence, the risk of the cash flow is unchanged. Still, because conservation of risk is violated the total cost of capital is only related to the asset risk before the capital structure is changed. After the change the total cost of capital is for some unknown reason suddenly higher than the asset risk since $r_2^{total} > r^A$. The risk of the cash flow now depends on how it is divided between equity and debt holders despite the fact that an investor holding all equity and debt both before and after receives the same total cash flow. We thus end up with a disconnect between the level of cash flow risk and the risk of the overall security portfolio.

5. Cost of Capital Fallacies

In this section, we illustrate some common cost of capital fallacies in the spirit of Admati et al. (2013). These fallacies rely on misconceptions of the underlying theories and are often used in the discussion of capital requirements.

A. Miscalculating the cost of capital requirements

We take as our starting point a debate which took place as a reaction to a recent decision by Danish authorities to introduce additional capital requirements on bank exposures to real estate companies (The Systemic Risk Council (2024)).

We first set up a common line of argumentation (see for example Kraka Advisory (2023) and Copenhagen Economics (2022)) used by banks to argue against the introduction of additional capital requirements. The argumentation, which explicitly assumes that risk conservation does not hold, is meant to illustrate that increased capital requirements are costly for the bank and hence its customers.

After setting up this argumentation we present an analysis of the same situation under the assumption that risk conservation does hold. We then compare the two results and explain the misconceptions underlying the analysis presented by the banking sector.

A.1. Costly Equity Capital (Incoherent Calculation)

The following argumentation illustrates according to the bank's point of view why increased capital requirements are costly. Before setting up the argumentation we stress again that the argumentation is not coherent and we explain why in the following sections.

Let us consider an entire banking sector as a whole and model it as a single bank. As part of the bank's asset portfolio, the bank has made loans for 548,000 to a specific sector/industry. The regulatory risk weight for this sector is 0.4. The regulatory capital requirement is now assumed to increase by 9 percentage points (of the risk weighted assets) for loans made to this sector. The required increase in equity capital is $548,000 \times 0.4 \times 0.09 = 19,728$. This is the amount of extra equity capital the bank needs in order to fulfill the increased capital requirements.

Now assume that the cost of equity capital (before the change in capital structure) is $r^E = 0.1021$. In order to satisfy this return requirement on the additional equity, the bank needs to return a yearly cash amount of $19,728 \times 0.1021 = 2,014$ to the new equity investors. Hence, this is the extra yearly cash flow that equity investors require on the equity invested into the bank. When we consider corporate taxes, the earnings before taxes needs to increase even more than the $2,014$ because the equity return requirement is on an after-tax basis. With a tax rate of 26% the increase in EBT needs to be $2,014/(1-0.26) = 2,722$. This is the increase in earnings before taxes that the bank needs to generate as an additional cash flow in order to satisfy the yearly return requirements for the additional equity capital.

Let us also assume that the additional equity substitutes debt in the capital structure. Hence, bank debt will be reduced by $19,728$. This then saves interest expenses of $19,728 \times 0.0305 = 602$, assuming that the cost of debt is $r^D = 0.0305$.

The bank argues that it only receives a so-called partial Modigliani-Miller effect when substituting debt for equity in the following sense: The bank gets an extra return requirement of $2,722$ per year on equity but saves 602 on debt interest payments. The net expense for the bank is therefore $2,014 - 602 = 1,413$. According to the bank they receive a 40% Modigliani-Miller effect. The interpretation is that 40% of the net expense will be covered by adjustments in the cost of capital for equity and debt but the remaining 60% will still be a net expense for the bank. Hence, the bank will have a reduction in the net expense of $0.40 \times 1,413 = 565$ from the so-called partial Modigliani-Miller effect. Hence the net expense for the bank becomes $1,413 - 565 = 848$.

The bank also argues that the internal cost distribution of the bank dictates that a given sector will have to cover their financing costs themselves. In other words, the sector that experiences an increase in capital requirement has to cover the net expense connected to the increased requirements. The sector in question in this case only makes up a fraction of the total loan portfolio for the bank. The total size of the bank is 3,822,510. This means that the sector makes up the fraction of $548,000/3,822,510 = 0.143$ of the total loan portfolio. The bank therefore splits up the savings from the so-called partial Modigliani-Miller effect so that the impacted sector gets the fraction 0.143 of the total savings. This is a saving of $565 \times 0.143 = 81$. The total net expense for this sector is then calculated as the total before tax equity requirement increase minus the reduction in interest payments and minus the fraction of the so-called partial Modigliani-Miller effect assigned to the sector. This amounts

Increased capital requirement	
$(548,000 \times 0.4 \times 0.09) =$	19,728
Additional equity cash flow needed	
$(19,728 \times 0.1021) =$	2,104
Corresponding pre-tax increase	
$(2,014/(1-0.26)) =$	2,722
Reduction in interest payments	
$(19,728 \times 0.0305) =$	602
Partial MM-effect	
$(0.4 \times (2,104-602)) =$	565
Sector saving	
$(565 \times 0.143) =$	81
Net expense for sector	
$(2,722-602-81) =$	2,039

Table I. Incoherent Cost of Capital Requirement Calculation

to a sector specific net expense of $2,722-602-81 = 2,039$ coming from the increased capital requirement. Table I summarizes the calculations.

In order to cover this additional net expense the customers of the sector will have to pay more in fees and interests on their loans, so that the bank each year can generate the additional cash flow of $2,039$ and thereby satisfy the return requirement from the new equity investors. Note that even with a full so-called Modigliani-Miller effect, there would still be a sizable net expense that the customers of the sector would need to cover according to the above argumentation. Redoing the same calculations as above but increasing the savings ratio to 100%, results in an net expense of $1,918$. Hence, there is a substantial cost even with a so-called full Modigliani-Miller effect.

The misleading conclusion from the above calculation is that increasing capital requirements will be costly and that this cost will have to be born by the customers of the sector for which the capital requirements originates. In addition, we can see that the assumptions about a so-called Modigliani-Miller effect is not of first order importance.

A.2. Coherent Calculation of the Costs

The frictions included in the above calculation are the tax shield and the so-called partial Modigliani-Miller effect. There is no mention of government support or a leverage ratchet effect. Hence, we will not include the latter two effects in the coherent calculation.

If we assume that conservation of risk holds then we know from section 2.B.2 and 3.A that a reduction of the leverage ratio will result in a value loss for the bank because of the lower tax shield. We also know however, that the total cost of capital will remain unchanged. The incoherent example does not assume that the cost of debt changes from before to after the change in capital structure and we will for simplicity keep this assumption. The outset of this coherent calculation is the reduction in the market value of debt. The interest payment will therefore decrease by $\Delta D \times r^D = 19,728 \times 0.0305 = 602$. The decrease in interest payments will increase the tax bill resulting in a value loss for the bank. The tax bill will increase with $602 \times 0.26 = 156$. The 156 is thus the yearly decrease in total after tax cash flows to investors. Note, for now, that this is far smaller than the cost of $2,039$ that the bank claims that it wants the customers of the sector to cover.

We can further illustrate how the argumentation used by the bank is incoherent with a numerical example. Assume that a bank has an unleveraged asset risk of $r^A = 0.05$. The assets generate a yearly EBIT of $70,000$. The bank is financed by debt and equity and the debt is issued at par value with a market value of 850,000 and a cost of $r^D = 0.0305$. The yearly interest payment is thus $r^D \times 850,000 = 25,925$. The earnings before taxes is then $EBT = 70,000 - 29,925 = 44,075$. With a tax rate of 26% the earnings (after tax) becomes $44,075 \times (1 - 0.26) = 32,615.5$. We can then calculate the total levered bank value as the discounted total cash flow available to investors

$$V^L = \frac{25,925 + 32,615.5}{0.05} = 1,170,810. \tag{67}$$

The equity value is the residual bank value and is given as $E = V^L - D = 320,810$. The market leverage ratio can then be calculated as $D/V^L = 0.274$. The cost of equity can be found as the cash flow to equity holders divided by the market value of equity

$$r^E = \frac{32,615.5}{320,810} = 0.10167. \tag{68}$$

The same number could also be found by using Equation (18), which gives

$$r^E = r^A + (r^A - r^D)\frac{D}{E} = 0.10167. \tag{69}$$

The calculations are summarized in Table II.

Next, assume that the bank reduces market value of debt by 50,000 to a value of 800,000. In this example, the reduction in debt does not decrease the cost of debt, which is still kept at $r^D = 0.0305$. We can then redo the calculation of the levered bank value. Interest payments now become $800,000 \times 0.0305 = 24,400$ and the earnings available to equity investors becomes $(70,000 - 24,400) \times (1 - 0,026) = 33,744$. Note that we have kept $EBIT$ unchanged

Total cost of capital r^A	0.05		
Cost of debt r^D	0.0305		
EBIT	70,000		
BEFORE			**AFTER**
Debt D	850,000	800,000	Debt D
Interest payments			Interest payments
$(850,000 \times 0.0305) =$	25,925	24,400	$= (800,000 \times 0.0305)$
EBT			EBT
$(70,000 - 25,925) =$	44,075	45,600	$= (70,000 - 24,400)$
Earnings (after tax)			Earnings (after tax)
$(44,075 \times (1 - 0.26)) =$	32,615	33,744	$= (45,600 \times (1 - 0.26))$
Bank value			Bank value
$((25,925 + 32,615)/0.05) =$	1,170,810	1,162,880	$= ((24,400 + 33,744)/0.05)$
Equity Value			Equity Value
$(1,170,810 - 850,000) =$	320,810	362,880	$= (1,162,880 - 800,000)$
Cost of Equity			Cost of Equity
$(32,615/320,810) =$	0.10167	0.09299	$= (33,744/362,880)$

Table II. Coherent Cost of Capital Requirement Calculation because the bank has not changed it's asset portfolio. The levered bank value now becomes

$$V^L = \frac{24,400+33,744}{0.05} = 1,162,880. \tag{70}$$

The discount rate is unchanged and equal to the expected asset return as explained in Section 3.A. The drop in bank value exactly equals the loss of tax shield. We can calculate the decrease in bank value

$$\Delta V^L = 1,170,810 - 1,162,880 = 7,930. \tag{71}$$

We can see that this is equal to the loss of tax shield by first calculating the decrease in interest payments, which is $25,925-24,400 = 1,525$. The tax bill therefore increases with $1,525 \times 0.26 = 396.5$ on a yearly basis. The present value of this tax bill increase becomes

$$\Delta PV(TS) = \frac{396,5}{0.05} = 7,930. \tag{72}$$

The total cost of capital is still $r^{\text{Total}} = r^A = 0.05$ as explained in Section 3.A and the cost of debt is assumed to be kept constant at $r^D = 0.0305$. Hence, the riskiness of the overall cash flow available to investors has not changed but the value of the bank has decreased. Since debt is now smaller, the cost of equity has decreased and the new cost of equity becomes

$$r^E = r^A + (r^A - r^D)\frac{D}{V^L - D} = 0.09299. \tag{73}$$

We can compare these results to the bank's incoherent calculation method. The bank would say that debt has decreased by 50,000. Equity thus needs to increase by 50,000 and since equity has a cost of $r^E = 0.10167$ then the bank needs to generate an additional cash flow after tax of $50,000 \times 0.10167 = 5,083$. On a pre-tax basis this would be $5,083/(1-0.26) = 6,869$. Now the savings on debt would be $50,000 \times 0.035 = 1,525$. Then we would also have the so-called 40% Modigliani-Miller effect of $(5,083 - 1,525) \times 0.40 = 1,423$. The total pre-tax cost would then be calculated as $6,869 - 1,525 - 1,423 = 3,921$. If we assume that this is a yearly cost and that we can discount it at the asset return then we have a present value of

$$\frac{3,921}{0.05} = 78,420. \tag{74}$$

We see that *the calculated loss of value becomes around ten times too large* compared to the coherent calculation above. The scale of the error is among other things a function of the initial leverage ratio in place.

We can also do the coherent calculation assuming a violation of the conservation of risk principle as described in Section 4. The bank reduces the market value of debt by 50,000 but maintains a cost of debt of $r^D = 0.0305$. This reduction in debt should decrease the expected equity return from $r^E = 0.10167$ to $r^E = 0.09299$. But because of a market failure the cost of equity is only reduced to $r^E = 0.10167 - 0.4 \times (0.10167 - 0.09299) = 0.0982$. We can then recalculate the value of equity by looking at the cash flow to equity, which is still 33,744 after the change in capital structure. The present value of this cash flow becomes

$$E^{\text{Violate}} = \frac{33,744}{0.0982} = 343,641. \tag{75}$$

This is compared to a value of 362,880 without a violation of the conservation of risk principle. Total bank value then becomes

$$E^{\text{Violate}} + D = 343,641 + 800,000 = 1,143,641. \tag{76}$$

This should be compared to the 1,162,880 without any violations. The total loss of bank value becomes 19,238, *which is still a factor four smaller than the incoherent calculation*.

For completeness, we note that the total bank value with violations, can also be found by recalculating the total cost of capital. The total cost of capital now becomes

$$r^{total} = \frac{E}{V} \times r^E + \frac{D}{V} \times r^D = \frac{343,641}{1,143,641} \times 0.0982 + \frac{80,000}{1,143,641} \times 0.0305 = 0.0508. \tag{77}$$

We can thus see that the total cost of capital has increased despite the fact that the asset risk has not changed, which is a violation of the conservation of risk. The total cash flow available to investors is the same as calculated under the conservation of risk example but it is for an unknown reason more risky now and we get

$$V^L = \frac{24,400 + 33,744}{0.0508} = 1,143,641. \tag{78}$$

To summarize, *the incoherent calculation overestimates the value loss by a factor ten* in this example. Even if we are willing to assume a violation of the conservation of risk then the incoherent calculation *overestimates the value loss by a factor four*. These value losses are calculated before the proclaimed split up and redistributing of the so-called partial Modigliani-Miller effect inside the divisions of the bank.

The two main fallacies in the incoherent example are:

Fallacy I: Partial Modigliani-Miller effect and Sector Distribution. The example contains a so-called partial Modigliani-Miller effect. This terminology is not completely clear, which is why we consistently use conservation of risk to refer to the principle underlying the calculations in Modigliani and Miller (1958). With a violation of the conservation of risk principle, it becomes difficult to do a coherent calculation because there is a mismatch between risk and expected return. When risk and expected return does not match, one can essentially assume anything and get any result. Still, note that having a partial effect is a recognition that the cost of equity becomes lower after the reduction in debt. However, in the example the tax calculation is still done with the pre-change cost of equity. This clearly seems incoherent.

The example also splits up the saving from the partial effect so that the sector in question only receives a fraction of the savings. In the incoherent example, the sector receives a saving of 81 out of a total of 565. The remaining $565 - 81 = 484$ is unaccounted for in the calculation. Unless the bank decrease fees and interests in the other divisions then this unaccounted for savings will be a net income for the bank and increase bank value.

Fallacy II: Increase in Sector EBIT. The incoherent calculation is based on the premise that the customers of a given sector will have to pay for the allocated bank equity capital. In the calculation EBT in the sector will be increased by 2,039 on a yearly basis.

This is a truly surprising assumption. If the bank was a monopolist and could set prices as they wanted to then why had the bank not already increased prices for the customers of the sector? If there is competition among banks then it is also unlikely that regulation would hit equally among all banks and make them simultaneously increase prices.

It is also unclear what the bank wants to obtain by increasing EBT. In the example, the increase is needed to meet the requirements of the new investors. But the value and return requirements for equity will adjust to the after regulation situation and the new equity return requirements will automatically be meet without adjusting EBT. The argument could instead be that the equity value decreases and prices are adjusted to maintain equity value at it's pre-regulation level.

There is however a fundamental difference between a need to increase earnings in order to satisfy a new equity requirement and increasing EBT in order to make sure that the old equity investors will not experience a value loss. Here it is also worth mentioning that the loss in value under conservation of risk is a private loss for the equity holders and not a social loss because the loss comes from a reduction in tax savings which can then be redistributed elsewhere in the economy (and likely at a higher value according to Admati and Hellwig (2013)).

Finally, if the banks increase prices in a given sector as assumed in the incoherent calculation, then it also needs to consider what happens to the risk of the cash flow coming from that sector. The calculation does not mention that risk would increase (besides the so-called partial Modigliani-Miller effect). However, if the same customers now need to pay more on their loan, it is reasonable to assume that credit risk would increase and this would have an effect on the overall asset risk of the bank. This effect is not explicitly discussed in the example.

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